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New Logical Connectives in Fuzzy Logic

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Outline

1 Who am I?

2 Motivation

3 Fuzzy Sheffer stroke

4 Done Work and Future Work

Who am I?

The bottom of the slide features two overlapping blue shapes. On the left, a light blue triangle points downwards. On the right, a darker blue shape, resembling a trapezoid or a wide triangle, points upwards. These shapes meet at a diagonal line that slopes downwards from left to right.

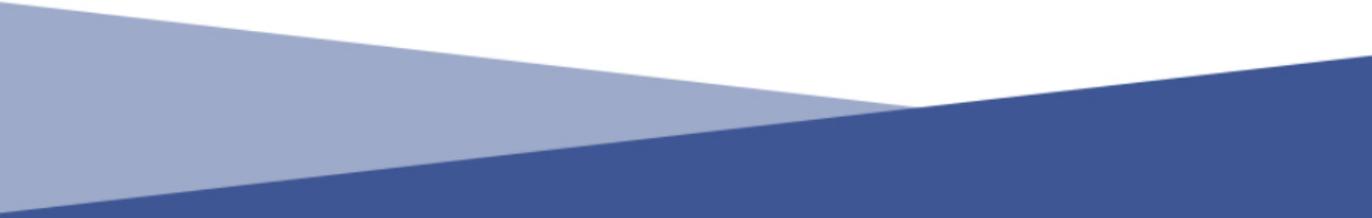
Who am I?

- Pedro Berruezo Guillamón.
- Assistant Lecturer at the University of the Balearic Islands (UIB).
- PhD Student at the UIB.

About my work...

- I am doing my master's thesis about:
Fuzzy Clustering Algorithms for Large-Scale Datasets.
- I am doing my Thesis about almost not studied fuzzy connectives.

Motivation

The bottom of the slide features a decorative graphic consisting of two overlapping blue shapes. The shape on the left is a light blue triangle pointing downwards. The shape on the right is a darker blue shape that also points downwards, overlapping the first one.

Refrigerator Alarm

Let us consider a refrigerator with an alarm and two sensors.

The sensors measure:

- The opening degree of the door.
- The internal temperature.



Refrigerator Alarm

Let us consider a refrigerator with an alarm **with different intensities** and two sensors.

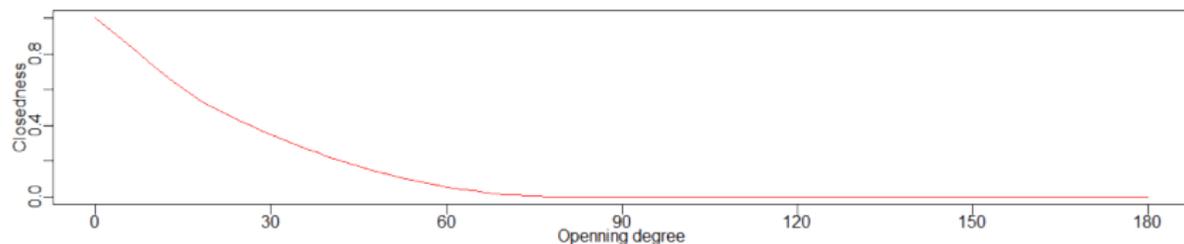
The sensors measure:

- The opening degree of the door.
- The internal temperature.



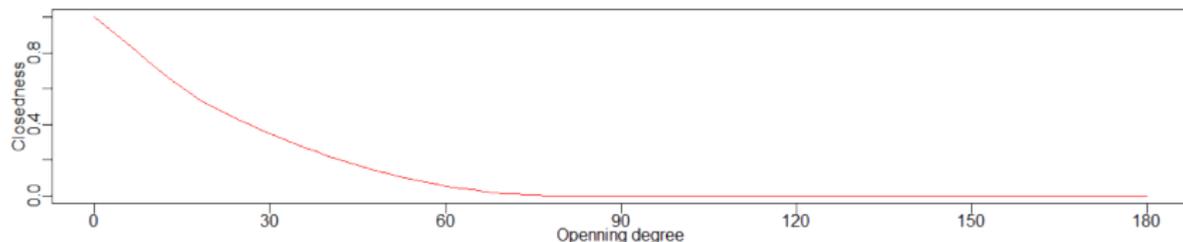
Refrigerator Alarm

- Let $A: X \rightarrow [0, 1]$ be the fuzzy set where X is the set of possible opening angles of the door. (**Closedness**)

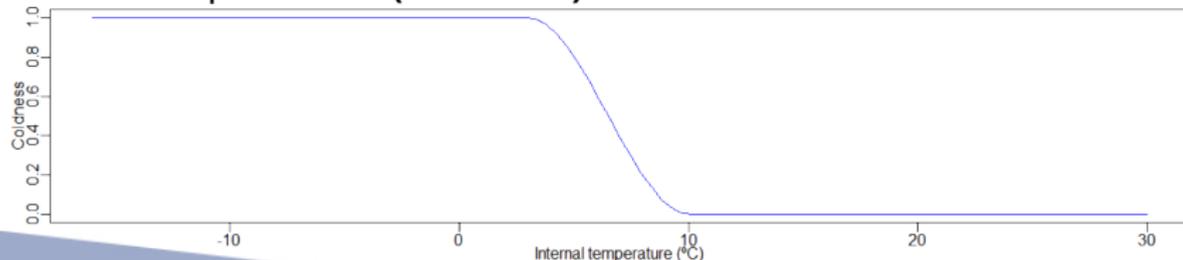


Refrigerator Alarm

- Let $A: X \rightarrow [0, 1]$ be the fuzzy set where X is the set of possible opening angles of the door. (**Closedness**)



- Let $B: Y \rightarrow [0, 1]$ be the fuzzy set where Y is the set of the possible internal temperatures. (**Coldness**)



Refrigerator Alarm

- Let $A: X \rightarrow [0, 1]$ be the fuzzy set that represents **Closedness**.
- Let $B: Y \rightarrow [0, 1]$ be the fuzzy set that represents **Coldness**.

We can model the alarm with the operator

$$F: [0, 1] \times [0, 1] \rightarrow [0, 1]$$

- If $F(a, b) = 0$ the alarm does not sound.
- Whenever $F(a, b)$ increases, so does the intensity of the alarm.

Refrigerator Alarm

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- If the internal temperature of the fridge is hot, $B(y) = 0$, then the alarm should sound, so $F(a, 0) = 1$ for all $a \in [0, 1]$.

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- If the door is open, $A(x) = 0$, then the alarm should sound, so $F(0, b) = 1$ for all $b \in [0, 1]$.
- If the internal temperature of the fridge is hot, $B(y) = 0$, then the alarm should sound, so $F(a, 0) = 1$ for all $a \in [0, 1]$.
- When the door opening angle decreases, so does the intensity of the alarm. That is,

$$\text{if } a_1 \leq a_2, \text{ then } F(a_1, b) \geq F(a_2, b) \text{ for all } b \in [0, 1].$$

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- If the door is open, $A(x) = 0$, then the alarm should sound, so $F(0, b) = 1$ for all $b \in [0, 1]$.
- If the internal temperature of the fridge is hot, $B(y) = 0$, then the alarm should sound, so $F(a, 0) = 1$ for all $a \in [0, 1]$.
- When the door opening angle decreases, so does the intensity of the alarm. That is,

$$\text{if } a_1 \leq a_2, \text{ then } F(a_1, b) \geq F(a_2, b) \text{ for all } b \in [0, 1].$$

- On the other hand, when the internal temperature decreases, the intensity of the alarm also decreases. That is,

$$\text{if } b_1 \leq b_2, \text{ then } F(a, b_1) \geq F(a, b_2) \text{ for all } a \in [0, 1].$$

Sheffer stroke

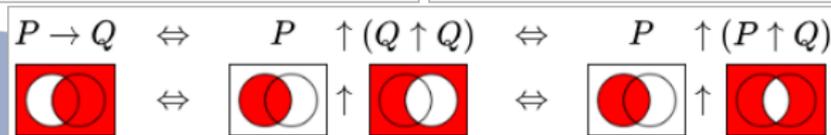
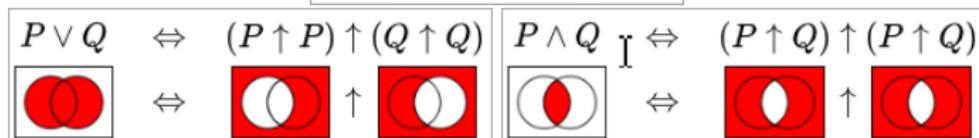
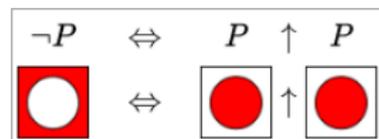
Table: Truth table for the classical Sheffer stroke.

p	q	$p \uparrow q$
0	0	1
0	1	1
1	0	1
1	1	0

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Definition

A function $SH: [0, 1]^2 \rightarrow [0, 1]$ is called a **fuzzy Sheffer stroke operation** (or fuzzy Sheffer stroke) if it satisfies, for all $x, y, z \in [0, 1]$, the following conditions:

- (SH1) $SH(x, z) \geq SH(y, z)$ for $x \leq y$, i.e., $SH(\cdot, z)$ is non-increasing,
- (SH2) $SH(x, y) \geq SH(x, z)$ for $y \leq z$, i.e., $SH(x, \cdot)$ is non-increasing,
- (SH3) $SH(0, 1) = SH(1, 0) = 1$ and $SH(1, 1) = 0$.

Examples of fuzzy Sheffer stroke

Example (The maximum fuzzy Sheffer stroke)

$$SH_{\max}(x, y) = \begin{cases} 0 & \text{if } (x, y) = (1, 1) \\ 1 & \text{otherwise} \end{cases}$$

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$$SH_{\min}(x, y) = \begin{cases} 0 & \text{if } (x, y) \in (0, 1]^2 \\ 1 & \text{otherwise} \end{cases}$$

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Example

$$SH_M(x, y) = \max\{1 - x, 1 - y\}$$

Done Work and Future Work

Conclusions

- We have generalized the Sheffer stroke operator to the fuzzy logic framework.
 - We have studied the different methods of construction of other fuzzy connectives from this one.
 - We have given some different construction methods to generate Sheffer strokes with different properties.
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Future Work

- Study the additional properties of the implications

$$I(x, y) = SH(x, SH(y, y))$$

and their intersection with other known implication families.

- Define, characterize and study the operator **Pierce Arrow**, and also its relationship with other connectives.
- Study the possible duality between fuzzy Sheffer Stroke and fuzzy Pierce Arrow.

A high-angle photograph of a stunning turquoise bay. The water is exceptionally clear, showing various shades of blue and green. The bay is framed by rugged, light-colored limestone cliffs with sparse green vegetation. Several people are seen swimming and wading in the shallow water. The sky is a clear, bright blue. The image is framed by dark blue geometric shapes at the top and bottom corners.

Grazie per la Tua Attenzione!