New Logical Connectives in Fuzzy Logic

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#SOM UIB

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Outline

1. Who am I?
2. Motivation
3. Fuzzy Sheffer stroke
4. Done Work and Future Work

Who am I?

Who am I?

- Pedro Berruezo Guillamón.
- Assistant Lecturer at the University of the Balearic Islands (UIB).
- PhD Student at the UIB.

About my work...

- I am doing my master's thesis about:

  Fuzzy Clustering Algorithms for Large-Scale Datasets.
- I am doing my Thesis about almost not studied fuzzy connectives.

Motivation

Refrigerator Alarm

Let us consider a refrigerator with an alarm and two sensors.

The sensors measure:

- The opening degree of the door.
- The internal temperature.

Refrigerator Alarm

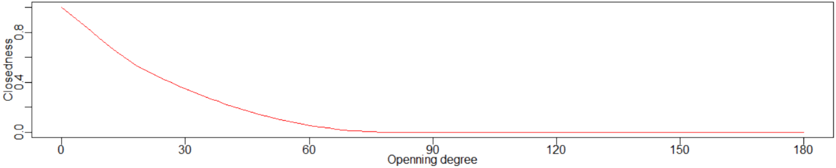
Let us consider a refrigerator with an alarm with different intensities and two sensors.

The sensors measure:

- The opening degree of the door.
- The internal temperature.

Refrigerator Alarm

- Let $A\colon X \to [0, 1]$ be the fuzzy set where X is the set of possible opening angles of the door. **(Closedness)**



Refrigerator Alarm

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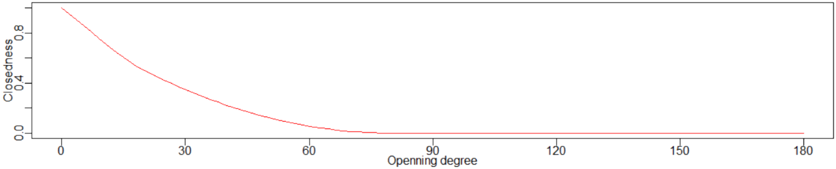
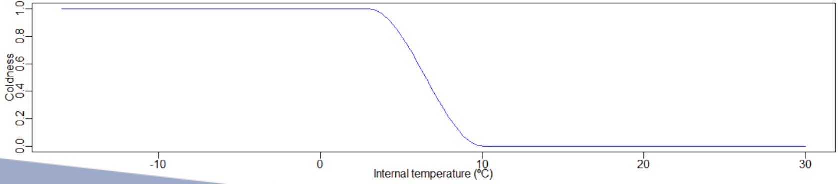


- Let $B\colon Y \to [0,1]$ be the fuzzy set where Y is the set of the possible internal temperatures. **(Coldness)**



Refrigerator Alarm

- Let $A\colon X \to [0,1]$ be the fuzzy set that represents **Closedness**.
- Let $B\colon Y \to [0,1]$ be the fuzzy set that represents **Coldness**.

We can model the alarm with the operator

$$F\colon [0,1] \times [0,1] \to [0,1]$$

- If $F(a,b) = 0$ the alarm does not sound.
- Whenever $F(a,b)$ increases, so does the intensity of the alarm.

Refrigerator Alarm

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- If the door is open, $A(x) = 0$, then the alarm should sound, so $F(0, b) = 1$ for all $b \in [0, 1]$.
- If the internal temperature of the fridge is hot, $B(y) = 0$, then the alarm should sound, so $F(a, 0) = 1$ for all $a \in [0, 1]$.

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- If the door is open, $A(x) = 0$, then the alarm should sound, so $F(0, b) = 1$ for all $b \in [0, 1]$.
- If the internal temperature of the fridge is hot, $B(y) = 0$, then the alarm should sound, so $F(a, 0) = 1$ for all $a \in [0, 1]$.
- When the door opening angle decreases, so does the intensity of the alarm. That is,

$$\text{if } a_1 \leq a_2, \text{ then } F(a_1, b) \geq F(a_2, b) \text{ for all } b \in [0, 1].$$

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- If the door is open, $A(x) = 0$, then the alarm should sound, so $F(0, b) = 1$ for all $b \in [0, 1]$.
- If the internal temperature of the fridge is hot, $B(y) = 0$, then the alarm should sound, so $F(a, 0) = 1$ for all $a \in [0, 1]$.
- When the door opening angle decreases, so does the intensity of the alarm. That is,

  $$\text{if } a_1 \leq a_2, \text{ then } F(a_1, b) \geq F(a_2, b) \text{ for all } b \in [0, 1].$$

- On the other hand, when the internal temperature decreases, the intensity of the alarm also decreases. That is,

  $$\text{if } b_1 \leq b_2, \text{ then } F(a, b_1) \geq F(a, b_2) \text{ for all } a \in [0, 1].$$

Sheffer stroke

Table: Truth table for the classical Sheffer stroke.

p	q	$p \uparrow q$
0	0	1
0	1	1
1	0	1
1	1	0

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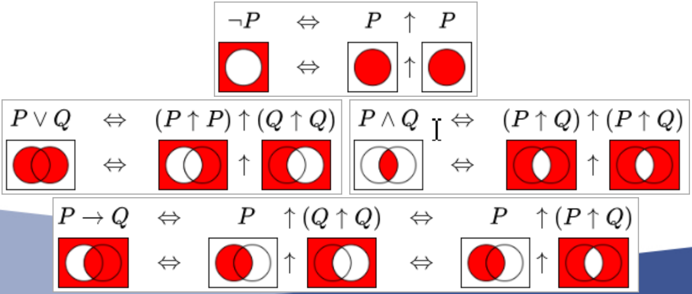

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Definition

A function $SH\colon [0,1]^2 \to [0,1]$ is called a **fuzzy Sheffer stroke operation** (or fuzzy Sheffer stroke) if it satisfies, for all $x, y, z \in [0,1]$, the following conditions:

(SH1) $SH(x,z) \geq SH(y,z)$ for $x \leq y$, i.e., $SH(\cdot, z)$ is non-increasing,

(SH2) $SH(x,y) \geq SH(x,z)$ for $y \leq z$, i.e., $SH(x, \cdot)$ is non-increasing,

(SH3) $SH(0,1) = SH(1,0) = 1$ and $SH(1,1) = 0$.

Examples of fuzzy Sheffer stroke

Example (The maximum fuzzy Sheffer stroke)

$$SH_{\max}(x,y) = \begin{cases} 0 & \text{if } (x,y) = (1,1) \\ 1 & \text{otherwise} \end{cases}$$

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$$SH_{\min}(x, y) = \begin{cases} 0 & \text{if } (x, y) \in (0, 1]^2 \\ 1 & \text{otherwise} \end{cases}$$

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Example

$$SH_M(x,y) = \max\{1-x, 1-y\}$$

Done Work and Future Work

Conclusions

- We have generalized the Sheffer stroke operator to the fuzzy logic framework.
- We have studied the different methods of construction of other fuzzy connectives from this one.
- We have given some different construction methods to generate Sheffer strokes with different properties.

Future Work

- Study the additional properties of the implications

$$I(x, y) = SH(x, SH(y, y))$$

  and their intersection with other known implication families.

- Define, characterize and study the operator **Pierce Arrow**, and also its relationship with other connectives.

- Study the possible duality between fuzzy Sheffer Stroke and fuzzy Pierce Arrow.

Grazie per la Tua Attenzione!